Endsem Prep: Engineering Mathematics I

Categorized highly-repeated questions from AISSMS PYQs (2024-2026).
Focus on these core patterns to maximize your exam score.

1. Mean Value Theorems (Rolle's & Lagrange's)

100% PROBABILITY

Every single paper features one question on Rolle's Theorem and one on Lagrange's Mean Value Theorem (LMVT) in Q1.

KEY FORMULAS

Rolle's Theorem: If $f(x)$ is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$.

Lagrange's Mean Value Theorem (LMVT): If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), there exists $c \in (a, b)$ such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

MOST REPEATED QUESTIONS

> Verify Rolle's theorem for polynomial functions like $f(x) = x^2 - 5x + 4$ on $[1, 4]$ or $f(x) = 3x^4 - 4x^2 + 5$ on $[-1, 1]$.

> Using LMVT, prove inequalities involving inverse trigonometric functions, such as: $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$.

2. Successive Differentiation (n^{th} Derivative)

100% PROBABILITY

Q1 always contains two questions on n^{th} derivatives: one requiring Partial Fractions, and one using Trigonometric transformations or Leibniz's Theorem.

KEY FORMULAS

Standard Form: $D^n\left(\frac{1}{ax+b}\right) = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

Trig Product Form: $D^n(e^{ax}\sin(bx + c)) = r^n e^{ax}\sin(bx + c + n\phi)$ where $r = \sqrt{a^2 + b^2}, \phi = \tan^{-1}(b/a)$.

Leibniz's Theorem: $(uv)_n = u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \cdots + u v_n$

MOST REPEATED QUESTIONS

> Find the n^{th} derivative of rational algebraic fractions like $y = \frac{x}{(x-1)(x-2)(x-3)}$ or $y = \frac{x}{x^2-10x+24}$.

> Find the n^{th} derivative of functions like $y = x^3\sin(4x + 3)$ or $y = x^2\cos(4x + 3)$.

3. Taylor's & Maclaurin's Expansion

100% PROBABILITY

Q2 consistently asks for a polynomial expansion using Taylor's Theorem and trigonometric/logarithmic expansions using Maclaurin's Theorem.

KEY FORMULAS

Taylor's Series (in powers of $x - a$):

$$f(x) = f(a) + (x - a)f'(a) + \frac{(x - a)^2}{2!}f''(a) + \ldots$$

Maclaurin's Series:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \ldots$$

MOST REPEATED QUESTIONS

> Using Taylor's theorem, expand a 3rd or 4th degree polynomial (e.g., $3x^3 - 2x^2 + x - 4$) in ascending powers of $(x + 2)$ or $(x - 2)$.

Apply Maclaurin's theorem to prove expansions for functions like $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ or $[\log(1+x)]^2$.

4. Euler's Theorem for Homogeneous Functions

100% PROBABILITY

Q3 is heavily dominated by Partial Differentiation, specifically applying Euler's theorem to first and second-order derivatives.

KEY FORMULAS

If u is a homogeneous function of degree n in x and y:

First Order: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

Second Order (Modified Euler's): If $f(u)$ is homogeneous of degree n, let $g(u) = \frac{n \cdot f(u)}{f'(u)}$. Then:

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)[g'(u) - 1]$$

MOST REPEATED QUESTIONS

If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

If $u = x^4 \sin\left(\frac{xy}{x^2+y^2}\right) + \frac{xy^2}{x+y}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

5. Jacobians & Functional Dependence

100% PROBABILITY

Every Q4 contains a question calculating a 3x3 Jacobian and testing for functional dependence.

KEY FORMULAS

Jacobian: $J = \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$

Functional Dependence: Functions u, v, w are functionally dependent if their Jacobian $J = 0$.

MOST REPEATED QUESTIONS

Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ given symmetric substitutions like $u = x+y+z, v = x^2+y^2+z^2, w = x^3+y^3+z^3$ or similar variants.

Examine for functional dependence for $u = \frac{x-y}{x+y}, v = \frac{x+y}{x}$ or similar sets.

6. Maxima/Minima & Errors/Approximations

100% PROBABILITY

The latter half of Q4 always features one optimization problem (max/min) and one real-world engineering percentage error calculation.

KEY FORMULAS

Maxima & Minima: Find stationary points where $p = \frac{\partial f}{\partial x} = 0$ and $q = \frac{\partial f}{\partial y} = 0$.
Calculate $r = f_{xx}, s = f_{xy}, t = f_{yy}$.
If $rt - s^2 > 0$ and $r > 0$ (Minimum). If $r < 0$ (Maximum).

Percentage Error:

$$\% \text{ Error in } u = \frac{\delta u}{u} \times 100 = \left(\frac{1}{u}\frac{\partial u}{\partial x}\delta x + \frac{1}{u}\frac{\partial u}{\partial y}\delta y\right) \times 100$$

MOST REPEATED QUESTIONS

Find the Maxima & Minima of algebraic surfaces like $x^3 + xy^2 - 3x^2 + 7$ or $x^2 + y^2 - 6x + 10$.

Find the percentage error in physical formulas (e.g., focal length $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, resistance $R = \frac{E}{I}$, or triangle area) given the % error in individual variables.